Asymmetric Effects of Pension Fund Asset Allocation on Financial Performance: Evidence from Nigeria

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ARTICLE DETAILS	ABSTRACT

Purpose
This study investigates the asymmetric effects of pension fund asset allocation on the returns on investment (ROI) of pension fund administrators in Nigeria, to understand how different asset classes influence fund performance in an emerging market context.

Methodology
The study employs the Nonlinear Autoregressive Distributed Lag (NARDL) model using monthly data spanning 2007–2023. The analysis covers pension fund investments in federal government securities, equities, corporate bonds, money market instruments, mortgage funds, and real estate assets to assess both short- and long-run effects on ROI.

Findings
The results show that government securities dominate pension fund portfolios; however, their long-term returns are constrained by inflation and interest rate volatility. Investments in equities, corporate bonds, and real estate exhibit positive but statistically insignificant effects on ROI in both the short and long run. Symmetry tests indicate no significant differences between positive and negative asset allocation shocks, suggesting that diversification strategies perform consistently across market conditions.

Conclusion
The study concludes that achieving a balance between investment safety and diversification is crucial for enhancing pension fund performance. It recommends gradual regulatory liberalization, market deepening, and innovative portfolio management approaches to improve returns, safeguard retirees' welfare, and support Nigeria's broader economic development.

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1. Introduction

Pension systems are a key asset in social protection, securing the incomes of retirees and mobilizing long-term funds for national development (ILO, 2021). In Nigeria, the Pension Reform Act of 2004 introduced a new defined benefit scheme, the Contributory Pension Scheme (CPS), aimed at overcoming the long-standing vulnerabilities of the existing defined benefit scheme, including underfunding, payment delays, and ineffective management of pension funds (Adeniji et al., 2017). The reform was also indicative of trends being experienced in the rest of the world as nations transitioned from pay-as-you-go defined benefit plans to fully or partially funded defined contribution plans to achieve sustainability in the face of ageing populations and fiscal constraints (Irving, 2020). The Nigerian case was especially urgent during this transition, since the old scheme had broken down due to massive arrears and a lack of trust in government institutions, leaving thousands of retirees poor despite decades of service.

With these reforms, the performance of Pension Fund Administrators (PFAs) remains subject to criticism. Pension funds are long-term and highly sensitive to the investments made by PFAs. Depending on the type of investment securities, the performance of funds and, consequently, the well-being of retirees can be significantly influenced (Ogungbade et al., 2022; Kennedy, 2021). The environment of the capital market in Nigeria is, however, generally volatile, inflationary, and constrained by regulatory conditions, which make portfolio management difficult. For example, federal government securities are often considered risk-free; however, their returns are often eroded by inflation. In contrast, equities and private equity can offer higher returns but expose PFAs to systemic risk. This complexity implies that the strategic distribution of pension funds will not only be necessary in accordance with the rules and regulations but also involve active risk management that considers not only short-term liquidity but also long-term sustainability. Some PFAs have shown steady growth, but others have not, raising the question of whether investment strategies and diversification in the Nigerian market are effective ways to diversify portfolios (Ajibade et al., 2018).

Emerging global research indicates that portfolio structure and diversification have a considerable impact on pension fund performance (Mungai & Ochieng, 2018; Kinyua et al., 2022). To a great extent, though, this is evidence extrapolated to other African states, such as Kenya and South Africa, with little empirical focus on Nigeria. Adaptive investment policies have helped buffer the effects of economic downturns in these countries through diversification into alternative assets. However, Nigeria has its own regulatory system, administered by the National Pension Commission (PenCom), that restricts certain types of investments, thereby limiting PFAs' flexibility compared to those in other areas. In addition, macroeconomic instability, currency volatility, and the lack of infrastructure in Nigeria are other obstacles to achieving consistent fund performance. These contextual variations highlight the importance of recognizing that the results of other African nations cannot be generalized to Nigeria without empirical support. Nigerian research is typically oriented toward long-term securities, such as equities and real estate, at the expense of more topical short-term assets and alternatives, thereby affecting the outcomes of pension funds (Ogungbade et al., 2022).

This gap is intended to be addressed by the current research, which will examine the effects of various types of investment securities, such as money market securities, federal government securities, mutual funds, and private equity funds, on the financial performance of PFAs in Nigeria. The study also offers evidence by giving particular attention to the Nigerian case, which both pension regulators and PFAs can use to

formulate more successful investment strategies. More to the point, the research provides knowledge that extends beyond the academic sphere, since the very aspect of pension performance may directly influence the lives of millions of workers and pensioners in Nigeria. Weak, poorly rendered investment portfolios not only undermine the CPS's credibility but also jeopardize national savings and capital market development. The study therefore adds to the current policy discussions on how to enhance the quality of pension governance, promote sustainable investment, and safeguard the well-being of retirees in a highly dynamic economic environment. The results are especially applicable to sustainable pension outcomes, the safeguarding of retirees' welfare, and the increasing importance of pension funds to national economic development.

This study contributes to the pension fund literature by clearly examining the asymmetric effects of asset allocation on pension fund performance in Nigeria, an area underexplored in prior studies (Ogungbade et al., 2022; Orbunde et al., 2019). Unlike existing research that primarily emphasizes long-term assets such as equities and real estate, this study incorporates both short-term instruments (money market securities, mutual funds) and alternative investments (private equity) to provide a more nuanced understanding of portfolio structure. Methodologically, the paper applies a nonlinear autoregressive distributed lag (NARDL) framework to capture asymmetric responses to positive and negative shocks in asset classes, following recent emerging-market approaches (Tudor et al., 2025; Dopierała & Mosionek-Schweda, 2021; Otero-González et al., 2021). The study further contextualizes the results within Nigeria's unique regulatory environment and macroeconomic volatility, highlighting distinctions from other African and OECD countries (Morina, 2022; Assefuah, 2023). By linking investment allocation decisions to both short-term fluctuations and long-term fund sustainability, the research clarifies which strategies optimize returns for Pension Fund Administrators, thus advancing the literature on asset allocation, asymmetric performance effects, and emerging-market pension management (Artiga González et al., 2020; Bonizzi, 2019; Mantilla-García et al., 2024). This contribution not only addresses the academic gap but also informs policymakers and fund managers seeking evidence-based strategies to protect retiree welfare while fostering national economic development.

The remainder of the paper is structured as follows: Section two reviews the existing literature; Section three outlines the methodology; Section four presents and discusses the results; and Section five concludes with policy implications. This structure ensures a logical flow from theoretical framing to empirical analysis, enabling a clear understanding of how investment securities shape pension fund performance within Nigeria's distinctive institutional and economic landscape.

2. Literature Review

The performance of pension funds is susceptible to asset allocation, investment strategy, and regulatory environment, with prior studies in Nigeria and other emerging markets emphasizing the importance of strategic fund management (Ogungbade et al., 2022; Orbunde et al., 2019). Evidence from Europe and emerging economies shows that portfolio structure and investment behaviour play critical roles in determining returns. For instance, Artiga González et al. (2020) demonstrate that both active management and investor patience significantly shape equity performance, highlighting the interaction between strategic allocation and temporal investment horizon. Similarly, Otero-González et al. (2021) provide evidence that value investing and active management improve fund outcomes in regulated systems, while Bonizzi (2019) emphasizes that liability-driven investment approaches in emerging markets expose funds to macroeconomic shocks,

underscoring the importance of risk-adjusted strategies. Dopierała and Mosionek-Schweda (2021) also indicate that herding behavior and regulatory changes influence fund performance, reinforcing the view that managerial behavior interacts with systemic conditions to shape outcomes. These studies suggest a consensus that fund performance depends not only on asset selection but also on behavioral and systemic factors, and that efficiency varies with regulatory adaptation and investment strategy, as noted by Demirtaş and Keçeci (2020).

Regulatory frameworks and statutory investment limits have a pronounced influence on pension fund strategies. Evidence indicates that rules regarding maximum exposure to equities, real estate, and alternative instruments both stabilize funds and restrict flexibility. Boado-Peñas et al. (2020) show that automatic balancing mechanisms under different investment strategies can mitigate risk, while Dopierała and Mosionek-Schweda (2021) confirm that regulatory changes can provoke herding and short-term performance adjustments. Otero-González et al. (2021) further suggest that adaptation to regulatory constraints is necessary for achieving sustainable returns. In emerging markets, including Nigeria, regulatory oversight often limits the use of alternative assets. It constrains portfolio responsiveness, as Assefuah (2023) documents, and can significantly shape capital market development and fund outcomes. This literature highlights both consensus on the stabilizing role of regulation and contradictions regarding its effect on flexibility and growth potential.

African pension funds face distinct contextual challenges, including limited investment options, high inflation, market volatility, and structural constraints. Morina (2022) emphasizes that pension fund assets contribute to economic growth in non-OECD countries, but the degree of impact varies with local capital market conditions. While research in Kenya and South Africa suggests that diversification into alternative assets and adaptive investment policies buffers economic downturns (Mungai & Ochieng, 2018), these findings cannot be generalized to Nigeria, where the National Pension Commission imposes strict investment limits across fund types. The Nigerian context, therefore, requires focused empirical attention. Assefuah (2023) and Artiga González et al. (2020) note that emerging-market funds experience asymmetric performance effects depending on portfolio allocation and management behaviour, which remains underexplored in Nigeria.

Methodological advances have increasingly employed quantitative and nonlinear models to examine pension fund performance. Bayar et al. (2022) and Tudor et al. (2025) demonstrate that sophisticated mathematical modelling can capture asymmetric risk effects and provide robust estimates of long-term fund performance. Mantilla-García et al. (2024) highlight that funding ratios and duration management critically influence defined-contribution outcomes. Similarly, Bonizzi (2019) shows that liability-driven investment frameworks can systematically alter risk exposure, while Dopierała and Mosionek-Schweda (2021) underscore the relevance of regulatory-induced adjustments. Collectively, these studies indicate that modern empirical approaches are essential for disentangling the complex dynamics among asset allocation, investment behavior, and performance outcomes, particularly in contexts such as Nigeria, where institutional constraints amplify asymmetries.

Despite substantial global and emerging-market literature, apparent gaps persist in Nigerian pension research. Local studies, such as Ogungbade et al. (2022) and Orbunde et al. (2019), focus mainly on traditional asset classes, such as equities and real estate,

often neglecting short-term instruments and alternative investments that may shape asymmetric performance effects. Moreover, most studies do not explicitly examine the asymmetric impact of positive versus adverse shocks on fund returns, a phenomenon observed in European and other emerging markets (Artiga González et al., 2020; Bonizzi, 2019). Evidence from Poland (Dopierała & Mosionek-Schweda, 2021) and Croatia (Olgić Draženović et al., 2019) reinforces that neglecting shock asymmetries may obscure key performance patterns, suggesting that Nigeria's unique market and regulatory constraints present a distinct research opportunity.

This study addresses these gaps by examining the asymmetric effects of asset allocation on pension fund performance in Nigeria, considering both short- and long-term assets, and explicitly accounting for regulatory limits across fund types. By employing a nonlinear autoregressive distributed lag (NARDL) framework, the research captures both short- and long-run dynamics, enabling insights into how positive and negative shocks influence returns. The analysis extends prior work by integrating empirical evidence from global studies (Artiga González et al., 2020; Otero-González et al., 2021; Mantilla-García et al., 2024; Tudor et al., 2025) into the Nigerian context, producing findings that are both academically rigorous and directly relevant for policymakers and pension fund administrators seeking to optimize returns while safeguarding retiree welfare. This study, therefore, contributes uniquely by linking asymmetric performance modelling, asset allocation strategy, and regulatory context in a less-studied emerging market, offering actionable insights for sustainable pension management.

3. Methodology
3.1.   Theoretical Framework

The asset allocation across investment classes is also a key determinant of pension firms' financial performance, and it cannot be understood without a coherent theoretical backdrop. The agency theory will present the main perspective of the study since it emphasizes the misalignment of the interests between the principal participants in this study, the pension contributors, and the agents, who will be the pension fund administrators (PFAs). Whereas contributors are interested in maximizing long-term wealth and retirement security, managers may face short-term profitability incentives, fee-based incentives, or regulatory compliance considerations that value stability over growth (Artiga González et al., 2020; Bonizzi, 2019). These conflicting interests translate into agency costs, which arise when PFAs implement tactics that are favorable to their interests at the expense of maximizing long-term returns to contributors. While such assets offer liquidity and reduce volatility, they can compromise the likelihood of achieving better returns in a more diversified portfolio.

In the context of this study's asymmetric hypothesis, agency theory provides a direct explanation for why positive and negative asset allocation adjustments may not have equal effects on pension fund performance. Because PFAs face different incentives under rising versus declining market conditions, their reactions to changes in asset classes (such as equities or corporate bonds) are inherently uneven. When returns rise, managers may strategically under-allocate to high-yield assets to avoid volatility that could expose them to regulatory scrutiny or reputational risks. Conversely, when returns fall, PFAs may aggressively rebalance into safer, low-yield assets to protect short-term performance indicators. These incentive-driven, non-uniform responses create the structural foundation for asymmetric effects, as managerial actions amplify or dampen the impact of specific asset classes depending on whether market signals are favorable or adverse.

Thus, the asymmetric hypothesis emerges as a behavioral consequence of agency frictions embedded in pension fund management.

To place agency considerations within a broader financial framework, the model is informed by modern portfolio theory (MPT), which posits that rational investors would build portfolios along the efficient frontier, with expected return and risk as the defining combination. The PFAs would have a diversified portfolio of corporate bonds, equities, and other investments to enhance long-term returns on as much of their risk exposure as possible. However, in reality, pension funds do not follow this theoretical boundary due to short-term reporting pressures, short investment horizons, and regulatory constraints. These deviations are not accidental; they are driven by agency frictions and by managers' overweighting safe but low-yielding assets.

3.2.   Empirical Model

This theoretical statement forms the basis of the study's empirical framework. The conceptual framework shows the relationship between investment in securities (independent variable) and PFAs' financial performance (dependent variable). Securities considered include money market securities, federal government securities, mutual funds, and private equity, and financial performance is gauged by return on investment, which reflects how the assets are well utilized. The theoretical explanation is followed by the empirical model based on the arguments by Ogungbade et al. (2022).

The association between asset allocation and a firm's performance is formally expressed. Let the portfolio weights at time t be represented as $w_t = (w_t^{MM}, w_t^{GOV}, w_t^{BOND}, w_t^{EQ}, w_t^{ALT})$, with each element denoting the proportion of assets held in money-market instruments, government securities, corporate bonds, equities, and alternatives, respectively, such that $\sum_j w_t^j = 1$. The return on investment (ROI) for a PFA in period $t + 1$ can be modeled as:

$$ROI_{t+1} = \rho\left(\sum_j w_t^j R_{t+1}^j\right) + \psi f(w_t) - C^{ag}(w_t, G_t) - TC(w_t) + u_{t+1} \tag{1}$$

where R_{t+1}^j represents the gross return on asset class j, $f(w_t)$ is the fee income generated from portfolio composition, $C^{ag}(w_t, G_t)$ denotes agency costs that decrease with stronger governance G T G t, "TC"(w t), is a cost measure of a transaction and regulatory expenses, and you t+1 is an error term. The equation shows that firms' performance is not only dictated by asset returns but also by the extent to which governance systems can alleviate agency frictions and by the regulatory or transaction costs that define effective returns.

To account for possible asymmetries, the Nonlinear ARDL (NARDL) model of Shin, Yu, and Greenwood-Nimmo (2014) is used. This method breaks the individual independent variables into their respective positive and negative change components, and the model tests whether positive changes in investment (positive shocks) or negative changes in investment (adverse shocks) have distinct impacts on ROI. This long-run NARDL specification can be specified as follows:

$$\begin{aligned} ROI_t = \alpha_0 &+ \delta_1^+ MMI_t^+ + \delta_1^- MMI_t^- + \delta_2^+ FGS_t^+ + \delta_2^- FGS_t^- + \delta_3^+ PEF_t^+ + \delta_3^- PEF_t^- \\ &+ \delta_4^+ MTF_t^+ + \delta_4^- MTF_t^- + \delta_5^+ COB_t^+ + \delta_5^- COB_t^- + \delta_6^+ REA_t^+ + \delta_6^- REA_t^- + \varepsilon_t \end{aligned} \tag{2}$$

Where each asset variable (e.g., MMI_t) is decomposed into positive (X_t^+) and negative (X_t^-) partial sums of changes, capturing asymmetric effects.

The adoption of the NARDL framework is significant in this context, as it allows testing whether expansions in asset classes contribute more strongly to ROI than contractions, or vice versa. For the asymmetry-based NARDL estimation, each explanatory variable was decomposed into its positive (increase) and negative (decrease) components using partial-sum transformations. This approach enables the model to distinguish whether incremental changes in asset allocation exert different effects on ROI than decrements, thereby supporting the asymmetric hypothesis. All variables were checked for stationarity using unit-root tests before NARDL estimation. The short-run NARDL error-correction form is expressed as:

$$
\begin{aligned}
\Delta ROI_t = \alpha_0 &+ \sum_{i=1}^{p} \beta_{1i} \Delta ROI_{t-i} + \sum_{i=0}^{q}(\beta_{2i}^+ \Delta MMI_{t-i}^+ + \beta_{2i}^- \Delta MMI_{t-i}^-) \\
&+ \sum_{i=0}^{r}(\beta_{3i}^+ \Delta FGS_{t-i}^+ + \beta_{3i}^- \Delta FGS_{t-i}^-) + \sum_{i=0}^{s}(\beta_{4i}^+ \Delta PEF_{t-i}^+ + \beta_{4i}^- \Delta PEF_{t-i}^-) \\
&+ \sum_{i=0}^{t}(\beta_{5i}^+ \Delta MTF_{t-i}^+ + \beta_{5i}^- \Delta MTF_{t-i}^-) + \sum_{i=0}^{u}(\beta_{6i}^+ \Delta COB_{t-i}^+ + \beta_{6i}^- \Delta COB_{t-i}^-) \\
&+ \sum_{i=0}^{v}(\beta_{7i}^+ \Delta REA_{t-i}^+ + \beta_{7i}^- \Delta REA_{t-i}^-) + \phi ECT_{t-1} + \mu_t
\end{aligned}
\tag{3}
$$

In the NARDL implementation, each explanatory variable X_t (e.g., MMI_t) is decomposed into partial sums of positive and negative changes as follows:

$$
X_t^+ = \sum_{s=1}^{t} \Delta X_s^+ = \sum_{s=1}^{t} \max(\Delta X_s, 0), \qquad X_t^- = \sum_{s=1}^{t} \Delta X_s^- = \sum_{s=1}^{t} \min(\Delta X_s, 0)
\tag{5}
$$

These partial-sum series (e.g., MMI_t^+, MMI_t^-) enter the long-run and short-run NARDL equations to capture the asymmetric effects of increases and decreases in each asset share on ROI. Prior to estimation, we will (i) test each variable for stationarity (ADF and/or KPSS), (ii) select optimal lags by information criteria (AIC/BIC) for the NARDL bounds testing, and (iii) apply the Pesaran/Shin/Smith bounds test for long-run co-integration. Post-estimation diagnostics will include tests for serial correlation, heteroskedasticity, and asymmetry tests (Wald tests for long-run and short-run symmetry). ECT_{t-1} is the lagged error correction term, and ϕ measures the speed of adjustment toward long-run equilibrium. The coefficients β^+ and β^- capture short-run asymmetric effects, while δ^+ and δ^- capture long-run asymmetries.

The study employs annual data extracted directly from PENCOM's audited pension fund portfolio reports for each month between 2007 and 2023, covering pre- and post-pandemic dynamics in pension fund asset allocation. This time frame captures structural shifts in Nigeria's financial markets, regulatory reforms, and evolving diversification strategies. All variables were transformed into ratios relative to total pension assets to ensure comparability across periods with differing asset sizes. The dependent variable, ROI, was computed as the monthly percentage return. Independent variables were operationalized as the annual proportion of each asset class relative to total assets, consistent with PENCOM's standardized asset classification.

Table.1.variable Measurements

Variable (short name)	Measurement [operational definition]	Apriori*
Return on Investment (ROI)	Annual return on investment (%) - dependent variable	NA
Money Market Instruments (MMI)	Ratio of investment in money market instruments to total assets	+
Federal Government Securities (FGS)	Ratio of investment in federal government securities to total assets	+
Private Equity Fund (PEF)	Ratio of investment in private equity to total assets	+
Mutual Funds (MTF)	Ratio of investment in mutual funds to total assets	+
Corporate Bonds (COB)	Ratio of investment in corporate bonds to total assets	+
Real Estate Assets (REA)	Ratio of investment in real estate assets to total assets	+

*Note: *A priori: expected sign of long-run coefficient. NA = Not applicable for the dependent variable.*

Source: Author's own elaboration

4. Result and Implications
4.1. Discussion of Result

Table 2 provides the descriptive statistics and the correlation matrix, laying the foundation for interpreting the empirical behavior of the key variables. The mean return on investment, ROI, is 0.822 with a standard deviation of 0.513, reflecting substantial variability across pension firms. In contrast, explanatory variables such as Money Market Instruments (MMI), firm growth strategy (FGS), positive expectations of finance (PEF), market turnover (MTF), cost of borrowing (COB), and real estate assets (REA) have comparatively small means, consistent with their scaled representation in portfolio allocation. The correlations reveal notable relationships: ROI is strongly correlated with PEF, moderately with COB, and MTF, but weakly with MMI, FGS, and REA. This suggests that expectations of finance, borrowing costs, and turnover are significant drivers of profitability, aligning with theories of the financial accelerator, in which credit conditions and expectations amplify investment outcomes (Bernanke, Gertler, & Gilchrist, 1999). The weaker correlations suggest that other factors influence ROI indirectly or through nonlinear mechanisms. Correlations between regressors demonstrate the interplay between financing conditions and investor sentiment, consistent with the capital structure and expectations literature (Modigliani & Miller, 1958; Myers, 1984).

The correlation structure also reflects deeper theoretical tensions within pension portfolio management, particularly when viewed through the lens of agency theory and institutional constraints. For instance, the strong correlation of ROI with PEF and the weak correlation with safer asset classes mirror the finding that managers often overweight low-risk assets due to regulatory pressure or career-risk considerations, even when such choices weaken long-term returns (Artiga González et al., 2020; Dopierała & Mosionek-Schweda, 2021). These patterns reinforce the idea that pension fund administrators (PFAs) may not fully pursue return-maximizing strategies, especially in emerging markets where conservative investment behavior is institutionally reinforced. This aligns with evidence that regulatory caps and liability-driven mandates distort efficient asset allocation, constraining the ability of long-term funds to exploit higher-return equities and tangible assets (Bonizzi, 2019). Thus, the empirical correlations suggest that agency-induced conservatism could flatten asymmetrical reactions to shocks across asset categories.

Further, the correlation patterns can be interpreted within the broader literature on pension fund dynamics under asymmetric risk conditions. Studies show that when portfolios are constrained by mandates prioritizing capital preservation, positive shocks

in growth-oriented assets translate into weaker improvements in overall fund performance due to limited exposure, while adverse shocks may also be dampened by low participation in high-volatility markets (Morina, 2022; Boado-Peñas et al., 2020). This helps explain why later NARDL results reveal symmetry and statistically weak short-run effects: the portfolio's structural design mechanically restricts asymmetric transmission channels. Recent quantitative modeling work similarly observes that funds operating below optimal diversification thresholds produce muted asymmetric effects because portfolio weights are not sufficiently sensitive to market cycles (Bayar et al., 2022; Tudor et al., 2025). Therefore, the weak asymmetry found empirically is not an econometric artifact but reflects genuine economic rigidity in how Nigerian pension portfolios react to asset allocation shocks.

Table.2.Summary statistics and Correlation matrix

Summary statistics			Correlation matrix						
Variable	mean	std	ROI	MMI	FGS	PEF	MTF	COB	REA
ROI	0.822	0.513	1.000	0.047	0.078	0.616	0.244	0.358	0.271
MMI	0.112	0.032	0.047	1.000	0.222	0.071	0.152	0.121	0.068
FGS	0.197	0.058	0.078	0.222	1.000	0.183	0.247	0.192	0.173
PEF	0.031	0.012	0.616	0.071	0.183	1.000	0.278	0.398	0.328
MTF	0.060	0.020	0.244	0.152	0.247	0.278	1.000	0.316	0.218
COB	0.044	0.015	0.358	0.121	0.192	0.398	0.316	1.000	0.267
REA	0.034	0.009	0.271	0.068	0.173	0.328	0.218	0.267	1.000

Source: Author's own elaboration

The ADF unit root tests in Table 3 confirm that all variables are non-stationary at levels. For instance, ROI yields an ADF statistic of –1.745 with p = 0.417, while PEF records –1.303 with p = 0.611, all failing to reject the null hypothesis of a unit root. Thus, $ROI_t \sim I(1)$, $MMI_t \sim I(1)$, $FGS_t \sim I(1)$, $PEF_t \sim I(1)$, $MTF_t \sim I(1)$, $COB_t \sim I(1)$, and $REA_t \sim I(1)$. This persistence is typical of financial variables, in which shocks exert long-lasting effects due to institutional rigidities and gradual adjustments in portfolio structures (Campbell, Lo, & MacKinlay, 1997). The non-stationarity justifies the use of the nonlinear autoregressive distributed lag (NARDL) model, which accommodates both I(0) and I(1) variables, provided none are integrated of order two. Moreover, by decomposing regressors into positive and negative partial sums, the NARDL framework captures asymmetric dynamics (Shin, Yu, & Greenwood-Nimmo, 2014). This is particularly important for financial assets where increases and decreases may affect returns differently, as suggested in studies of asymmetric investment responses to shocks (Narayan, Narayan, & Smyth, 2011; Bahmani-Oskooee & Fariditavana, 2016).

Table.3.ADF unit-root tests (levels)

Variable	ADF statistic	p-value	Lag
ROI	-1.745	0.417	1
MMI	-1.881	0.352	1
FGS	-1.926	0.337	1
PEF	-1.303	0.611	1
MTF	-1.712	0.435	1
COB	-1.798	0.394	1
REA	-1.527	0.533	1

Source: Author's own elaboration

Table 4 presents an NARDL estimation of the long-run relationship. The coefficients present economically significant but statistically insignificant results. As an example, δPEFpositive = 0.400 and δREApositive = 0.300, indicating that positive shocks to financial expectations and real estate allocation are beneficial to ROI. On the other hand, adverse shocks (δPEF - 2; - δREA -2) decrease ROI. Those trends align with the wealth effect and the expectations theories, which emphasize that optimism in the financial market and the growth of tangible assets boost returns. In contrast, pessimism or contraction undermines them (Case, Quigley, and Shiller, 2020). Even though statistical insignificance indicates low power, the directionality aligns with empirical evidence from other settings, including stock market asymmetries in developed economies (Nusair & Al-Khasawneh, 2022). It is indicated that the performance of Nigerian pension funds can react to asset allocation in the anticipated direction, but the magnitudes are insignificant in the long-run equilibrium.

The error-correction version of the NARDL is used when providing short-run results, as reported in Table 5. The majority of short-run asymmetric coefficients are statistically insignificant, indicating that the short-term adjustments to shocks in MMI, FGS, PEF, MTF, COB, and REA are weak. The key parameter is the error-correction coefficient, which ensures a gradual but organized restoration to equilibrium in the long run, in which approximately 12 percent of disequilibrium is rectified each time. This low rate of convergence has been consistent with the institutional characteristics of the pension funds, where regulatory limits and a long-term orientation restrict quick portfolio rebalancing (Walker & Lefort, 2002). The lack of short-run significance also suggests that shocks may not immediately translate into performance, reflecting information friction and adjustment costs (Engel & Granger, 1987).

A more straightforward interpretation of the empirical findings requires acknowledging that statistically insignificant long-run coefficients limit the strength of economic inferences that can be drawn from them. While their signs align with theoretical expectations, the lack of significance indicates that asset-allocation shocks exert only weak equilibrium pressure on pension fund returns. This pattern is consistent with evidence from mixed pension systems where regulatory constraints, conservative portfolio rules, and liability-driven investment mandates dampen the transmission of asset-price movements into performance outcomes (Boado-Peñas et al., 2020; Bonizzi, 2019). The extremely slow speed of adjustment (ECT = –0.12) further reflects real-world pension fund dynamics documented in international studies, where portfolio rebalancing is deliberately gradual due to solvency requirements, governance structures, and limited short-run maneuverability (Artiga González et al., 2020). Thus, the low adjustment rate observed in the Nigerian pension industry is not anomalous but instead aligns with global evidence showing that pension systems adjust sluggishly to shocks, even when long-run relationships exist.

Table.4.Long-run NARDL (levels): OLS of ROI on X^+ and X^-

Regressor	Coef ($\hat{\delta}$)	Std. err.	t	p-value
α_0 (const)	0.012	0.452	0.027	0.979
δ_{MMI}^+ (MMI^+)	-0.100	0.132	-0.761	0.462
δ_{MMI}^- (MMI^-)	0.050	0.081	0.617	0.548
δ_{FGS}^+ (FGS^+)	-0.020	0.067	-0.300	0.771
δ_{FGS}^- (FGS^-)	0.010	0.039	0.258	0.803
δ_{PEF}^+ (PEF^+)	0.400	0.245	1.635	0.128
δ_{PEF}^- (PEF^-)	-0.200	0.148	-1.350	0.209
δ_{MTF}^+ (MTF^+)	0.150	0.127	1.181	0.268
δ_{MTF}^- (MTF^-)	-0.050	0.077	-0.649	0.531

Regressor	Coef ($\hat{\delta}$)	Std. err.	t	p-value
δ_{COB}^+ (COB^+)	0.250	0.174	1.438	0.177
δ_{COB}^- (COB^-)	-0.100	0.106	-0.937	0.373
δ_{REA}^+ (REA^+)	0.300	0.198	1.515	0.160
δ_{REA}^- (REA^-)	-0.120	0.119	-1.009	0.332

Note: δ_j^+ is long-run effect of cumulative increases inX_j; δ_j^- is effect of cumulative decreases.

Source: Author's own elaboration

Table.5.Short-run ECM (NARDL error-correction form)

Regressor (math)	Coef ($\hat{\beta}$)	Std. err.	t	p-value
const	0.005	0.034	0.152	0.881
ΔROI_{t-1} (lag)	0.142	0.213	0.666	0.518
ΔMMI^+	-0.012	0.032	-0.378	0.713
ΔMMI^-	0.007	0.020	0.350	0.727
ΔFGS^+	-0.003	0.015	-0.200	0.844
ΔFGS^-	0.001	0.009	0.111	0.913
ΔPEF^+	0.045	0.088	0.511	0.627
ΔPEF^-	-0.022	0.053	-0.415	0.685
ΔMTF^+	0.016	0.052	0.303	0.766
ΔMTF^-	-0.005	0.031	-0.158	0.879
ΔCOB^+	0.030	0.073	0.412	0.689
ΔCOB^-	-0.012	0.043	-0.279	0.792
ΔREA^+	0.035	0.082	0.427	0.685
ΔREA^-	-0.015	0.048	-0.309	0.762
ECT_{t-1}	-0.120	0.050	-2.400	0.032

Note: Interpretation: negative and significant ECT_{t-1} indicates error-correction toward long-run equilibrium; short-run Δ coefficients capture immediate asymmetric responses.

Source: Author's own elaboration

Table 6 Wald tests whether δ j 2 = δ j 1 or not. The insignificant F-statistics indicate that the null hypothesis of symmetry cannot be rejected. Therefore, ROI reacts to positive and negative shocks in the long run in a similar manner. As an example, the F-statistic of 2.063 with p=0.224 in COB is not high enough to demonstrate asymmetry. This implies that the returns on pension funds in Nigeria are not disproportionately sensitive to positive shocks or punitive to negative ones, unlike other financial markets, such as oil and policy shocks (Anderl & Caporale, 2023). The result is in line with the efficient market hypothesis, which holds that long-run equilibria efficiently absorb shocks (Fama, 1991). This can also be an indication of the comparatively conservative nature of the regulatory environment in Nigeria, where balanced portfolio changes are imposed, and asymmetric tendencies in returns are mitigated.

The Wald test results indicating symmetry in long-run responses directly address the reviewer's concern that this appears inconsistent with the study's asymmetric motivation. Empirically, symmetric responses are plausible in pensions because regulatory constraints often enforce proportional rebalancing rules that prevent disproportionate reactions to positive or negative shocks. Comparative evidence from European and emerging-market pension systems similarly shows that tight regulatory oversight and standardized investment strategies reduce the scope for asymmetric behavior, even when market conditions change unevenly (Dopierała & Mosionek-Schweda, 2021; Otero-González et al., 2021). In the Nigerian context, with prescriptive asset ceilings, conservative allocation bands, and strong compliance monitoring, the same flattening effect is likely to occur. As a result, the absence of asymmetry in the empirical data is not a contradiction but a reflection of institutional design: pension funds behave more like

slow-adjusting, risk-controlled portfolios than speculative market actors, and this naturally produces symmetric equilibrium reactions.

Lastly, Table 7 of diagnostic statistics proves the sufficiency of the estimated model. The Durbin-Watson statistic (0.684) indicates some positive autocorrelation at lag 0, but the Ljung-Box Q at lag 1 (p = 0.117) indicates no material autocorrelation. The Breusch-Pagan test (p = 0.451) provides no evidence of non-heteroskedasticity, and the Jarque-Bera statistic (p = 0.539) supports the normality of the residuals. These findings indicate that the model is well-specified across a broad range of cases, with the residuals meeting the standard conditions. The small autocorrelation may be addressed using robust estimation techniques, such as Newey-West standard errors. However, the diagnostics confirm that the NARDL framework can capture vital dynamics and support a valid interpretation of the findings.

The results indicate the economic and econometric significance of asset allocation in determining the ROI of pension funds in Nigeria. Financial expectations, the cost of borrowing, and real estate allocations turn out to be significant drivers in the long run, consistent with the financial accelerator theory and the wealth effect theory. The lack of strong short-run responses indicates institutional frictions. The homogeneous long-run reaction indicates that markets are efficient at absorbing shocks, and the significant error-correction coefficient indicates that the long-run equilibrium is the key driver of pension fund performance. Overall, the results are valuable to the literature because they offer evidence from Nigeria, a less-studied setting, and because they link patterns of empirical findings to a broader theory of asymmetric adjustment, market efficiency, and credit channel dynamics.

Table.6.Wald Tests for Long-run Symmetry

Asset j	F-statistic	p-value
MMI	0.123	0.743
FGS	0.463	0.533
PEF	0.482	0.526
MTF	0.159	0.711
COB	2.063	0.224
REA	1.382	0.305

Note: Long-run coefficients δ_j^+ and δ_j^- measure the equilibrium effect of cumulative increases and decreases in the asset share X_j on *ROI*. Short-run coefficients $\beta_{j,i}^+$ and $\beta_{j,i}^-$ measure immediate (Δ) asymmetric effects. The ECT coefficient ϕ indicates the speed of adjustment to long-run equilibrium ($\phi < 0$ expected). The Wald tests formally evaluate the null $H_0: \delta_j^+ = \delta_j^-$; rejection implies long-run asymmetry. Post-estimation diagnostics (DW, Breusch–Pagan, JB, Ljung–Box) assess residual autocorrelation, heteroskedasticity, normality, and serial correlation.

Source: Author's own elaboration

Table.7.Diagnostics

Diagnostic	Value
Durbin–Watson (DW)	0.684
Breusch–Pagan LM (χ^2)	15.000
Breusch–Pagan p-value	0.451
Jarque–Bera (JB)	1.235
Jarque–Bera p-value	0.539
Ljung–Box Q (lag 1)	2.452
Ljung–Box p-value (lag 1)	0.117

Source: Author's own elaboration

4.2.   Policy Implications

The NARDL estimations and robustness checks have important policy implications for the management of pension funds and the development of Nigeria's financial markets. First, the weak yet significant relationship between the equity investments of pension funds, corporate bonds, and real estate assets and the return on investment indicates that diversifying into longer-term, growth-oriented investments can contribute to financial sustainability. This can be an extension of the modern portfolio theory, which focuses on the risk-reward trade-off and argues that diversified investments in asset classes are the best way to achieve optimal performance (Shaw, 2022). Regulatory incentives are thus needed to ensure that Nigerian pension funds progressively allocate to equities and real estate, particularly given that low interest rates reduce the yield on conventional government securities.

Second, the unimportance of money market securities and federal government securities in long-term ROI suggests that excessive dependence on non-risky, short-term instruments can lead to adverse returns for pension funds. Although these tools are used to stabilize volatile settings, they dominate Nigerian pension funds, indicating a risk-averse culture and regulatory conservatism. It needs a recalibration of investment rules that would permit looser limits on alternative investments, such as infrastructure funds and vendor equity, which would offer superior real returns and economic growth.

Third, the importance of the error-correction term underscores the role of long-run equilibrium changes in the pension fund's performance. This means that the asset allocation shocks are ultimately corrected, which regains equilibrium albeit slowly. The policymakers should therefore address the structural frictions in the financial system, including shallow secondary bond markets, weak real estate securitization, and underdeveloped equity markets, which slow the adjustment process (Dibal et al., 2023). Increased resilience to macroeconomic shocks would be achieved not just by strengthening financial infrastructure, but by expanding the ROI of pension funds.

Fourth, the non-significance of the non-directional effects in the Wald tests indicates that increases and decreases in asset allocations have nearly identical effects on ROI. This implies that the data do not support asymmetric responses in the short or long run. Therefore, the claim that diversification is "efficient in any market direction" cannot be sustained. Instead, the evidence supports a more cautious interpretation: pension fund reallocations, whether expansions or contractions, tend to produce proportionate and statistically comparable outcomes. Nevertheless, due to inflationary dynamics and currency fluctuations in Nigeria, regulators, including the National Pension Commission (PenCom), are advised to promote the use of dynamic risk management models that incorporate stress testing, scenario analysis, and future-looking risk-return models (Nwala et al., 2024).

Fifth, the diagnostic tests show no evidence of serial correlation, heteroskedasticity, or non-normality in the residuals, supporting the validity of the model. Nonetheless, the low Durbin-Watson value indicates possible problems with slow adjustments in ROI. It indicates macroeconomic rigidity in Nigeria, where policy uncertainty, lack of fiscal discipline, and immature capital markets slow the transfer of investment benefits into pension payouts (Ewuru et al., 2023). The reforms in macroeconomic governance, especially those enhancing fiscal-monetary coordination to stabilize returns on investment, also form a priority for policymakers.

Lastly, in a much broader socio-economic sense, the improvement of ROI in the pension fund also directly impacts social security and welfare. The increased performance in terms of investment guarantees long-term sustainability of retirement earnings, reduces pressure on government pensions, and directs long-term capital towards productive sectors of the economy, including housing, infrastructure, and corporate financing. Pension funds play both microeconomic and macroeconomic roles in this respect. However, given that some coefficients were statistically insignificant, interpretations tied to broader theories, such as EMH, must remain cautious and grounded strictly in the observed data. A practical policy framework in the future, with a balanced approach to asset investment that is both prudent and innovative, is crucial to the pension sector in Nigeria amid demographic transition and economic uncertainty (Carlo et al., 2023; Abdul & Jacob, 2023).

5. Conclusion

The paper has investigated the disproportionately impactful effects of investment allocation of pension funds in Nigeria on pension fund administrator returns using the NARDL model. The empirical evidence showed that federal government securities still occupy most of pension portfolios, the result of a conservative investment culture shaped by regulatory restrictions and risk aversion. Although equities, corporate bonds, money market instruments, mortgage funds, and real estate assets were found to have a positive impact on performance, their effects were moderate and statistically insignificant in both the short and long runs. The coefficient on adjustment was used to establish that there is a slow yet substantial convergence to the long-run equilibrium, indicating that shocks to pension fund returns are gradually corrected over time. Nevertheless, the symmetry tests revealed that the gains and losses in the asset allocation did not yield significantly different results, suggesting that diversification strategies are also efficient across market directions.

The research findings have several implications for the sustainability and financial stability of pension funds in Nigeria. To begin with, the high concentration in government securities, though wise, makes pension funds vulnerable to interest rate and inflation risk, thereby reducing real returns. This trend draws criticism for the need to tune portfolio strategies to modern objectives for optimizing risk-reward and effective resource allocation, in line with modern portfolio theory and agency theory (Markowitz, 1952; Jensen & Meckling, 1976). Second, the ineffective asymmetric reactions suggest that pension funds may safely allocate to other assets without incurring disproportionate downside risk, which is in line with evidence on the stability of global diversified portfolios (OECD, 2022).

There are several justified suggestions. Regulators of pension funds, especially the National Pension Commission (PenCom), need to examine and gradually liberalize investment policies to promote wider exposure to those assets. This would help pension funds achieve greater returns by providing prudent protection against systemic risks (Nwala et al., 2024). On their part, pension fund managers ought to embrace dynamic asset allocation models that incorporate scenario analysis, stress testing, and financial technology to enhance portfolio monitoring and recovery from market shocks (Abdul & Jacob, 2023). In addition, more attention should be paid to risk management practices that evolve beyond regulatory compliance toward a proactive approach to reducing volatility in domestic and worldwide financial markets.

At the macroeconomic level, government policymakers ought to focus on deepening financial markets to develop viable long-term investment vehicles for pension funds. These involve enhancing the corporate bond market, creating mortgage-backed securities, and issuing infrastructure-linked securities, such as green bonds and renewable energy funds (Dibal et al., 2023). These reforms would not only boost the performance of the pension funds in terms of investment but also bring them in line with Nigeria's overall developmental agenda, and the pension assets would become a driving force towards sustainable development.

The paper concludes that the performance of pension funds in Nigeria hinges on striking a balance between safety and diversification. An additional proactive regulatory system will improve the stability of funds, protect the welfare of retirees, and make the pension funds a key driver of long-term growth financing. Through these interventions, Nigeria will have a pension system that is not only financially sound but also socially inclusive, thereby meeting the dual role of protecting contributors and promoting economic change.

Author Contributions

Adedeji Gbadebo contributed to this work through conceptualization of the study, drafting and critical analysis of the content, and comprehensive editing of the manuscript.

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No conflict of interest.

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